Blind Equalization Based On Blind Separation with Toeplitz Constraint

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Abstract—**Blind equalization (BE) has been modeled as a blind source separation (BSS) problem and achieved using BSS algorithms. One drawback of on-line algorithms considered in previous work for blind equalization is their slow convergence. We show that the Toeplitz structure of the mixing matrix and the separating matrix in the BSS model for BE can be exploited to obtain faster convergence and better performance. In addition, a constraint on the length of the equalizer impulse response can provide further improvement. For sources with independent in-phase and quadrature parts, we can use I/Q constrained BSS technique to do equalization and achieve phase recovery. We use the well-known equivariant adaptive source separation via independence (EASI) algorithm to illustrate the ideas, although the approach we describe is more generally applicable.**

I. INTRODUCTION

Blind source separation (BSS) is the problem of recovering independent sources from observed mixtures when no information about the mixing process and no training sequence is available. Based on the theory of independent component analysis (ICA), several BSS algorithms have been proposed and they have been applied in a wide range of applications [1], [2]. One application is Blind Equalization (BE). Since inter-symbol interference (ISI) produces channel outputs that are mixtures of independent input symbols, BE can be formulated as a BSS problem and achieved using BSS algorithms [3], [4].

A well-known BSS algorithm that we will use is the equivariant adaptive source separation via independence (EASI) algorithm [5]. As is the case for most on-line adaptive BE algorithms and in particular for BSS approaches, convergence of the algorithms can be slow. In this work we exploit the Toeplitz structure of the BSS mixing matrix and the separating matrix for the BE problem. The BSS separating matrix structure can be constrained in the adaptive updates for the equalizer. This approach will improve convergence rate, and enhance equalization performance. Furthermore, since the equalizer coefficients are contained in the separating matrix, we can use any reasonable length constraint on the equalizer impulse response to obtain further improvements in performance.

In BSS it is known that sources can be recovered up to a complex scalar and with position-order ambiguity. In [6] and [7], an I/Q constraint is considered to separate the sources and recover the phase at the same time. It is shown in [6] that with the I/Q constraint, ordering indeterminacy and $\pi/2$ phase ambiguity still remain. In our work, due to the Toeplitz structure of the mixing matrix, there will be no order or phase ambiguity.

Our work will illustrate the ideas by first setting up a specific BE scenario in which transmission takes place as a sequence of zero-padded blocks of symbols. We will show that convergence speed-up and improved performance is also obtained with an uninterrupted sequence of transmitted symbols without a block structure. Simulation results will illustrate the performance improvements that are possible using these ideas.

II. PROBLEM FORMULATION AND EASI ALGORTIHM

A. Problem formulation

Let $\boldsymbol{s}$ be an $n\times 1$ vector of independent outputs from n sources, A be an $m\times n$ mixing matrix, and let $\boldsymbol{x}$ be the vector of observations so that

$$\boldsymbol{x} = A\boldsymbol{s}. \tag{1}$$

The objective of BSS is to get a separating matrix B such that

$$\boldsymbol{y} = B\boldsymbol{x} \tag{2}$$

is an estimate of $\boldsymbol{s}$, i.e. $C \triangleq BA \approx I$. In standard BSS, it is assumed that matrix A is full rank with $m \geq n$.

First consider an equalization problem for a system with inserted zeros between transmitted signal blocks. Suppose $\boldsymbol{h} = [h(0), h(1), ..., h(L)]$ is the impulse response of length $L+1$ of an FIR causal channel. Source symbols are transmitted through the channel in blocks of size P, padded by L zeros at the beginning of each block. The total number of symbols to be transmitted in each block is thus $N=P+L$. Denote the k-th transmitted block of symbols (of length N) as $\boldsymbol{d}_k = [a_k(P), a_k(P-1), ..., a_k(1), 0, ..., 0]^T$, where $a_k(j)$, $j = 1, 2.., P$ is the j-th symbol in the k-th block. With additive Gaussian noise, the k-th vector of observations of length P is

$$\boldsymbol{x}_k = H\boldsymbol{d}_k + \boldsymbol{v}_k \ . \tag{3}$$

Here H is the $P\times N$ mixing matrix composed of the channel impulse response, i.e.

$$H = \begin{pmatrix} h(0) & h(1) & \dots & h(L) & \dots & \dots & 0 \\ & \ddots & & \dots & & \ddots & \dots \\ \dots & h(0) & h(1) & \dots & h(L) & \dots & \\ & & \ddots & & \dots & & \ddots \\ 0 & \dots & \dots & h(0) & h(1) & \dots & h(L) \end{pmatrix} \tag{4}$$

and $\boldsymbol{v}_k$ is the noise vector of the k-th block.

B. EASI algorithm

Denote the separating matrix at the k-th iteration as B_k. The normalized EASI algorithm has the updating rule:

$$B_{k+1} = B_k - \lambda\left[\frac{\boldsymbol{y}_k\boldsymbol{y}_k^T - I}{1+\lambda\boldsymbol{y}_k^T\boldsymbol{y}_k} + \frac{\boldsymbol{g}(\boldsymbol{y}_k)\boldsymbol{y}_k^T - \boldsymbol{y}_k\boldsymbol{g}^T(\boldsymbol{y}_k)}{1+\lambda\,|\,\boldsymbol{y}_k^T\boldsymbol{g}(\boldsymbol{y}_k)\,|}\right]B_k \quad , \qquad (5)$$

where $\boldsymbol{y}_k = B_k\boldsymbol{x}_k$, λ is the adaptation step size, and $\boldsymbol{g}(\cdot)$ is a component-wise nonlinear odd function. In this updating scheme the numerator of the first term in brackets effects a whitening constraint while the numerator of the second term effects a nonlinear decorrelation for independence. The denominator normalizations control the effective step sizes.

III. BE VIA BSS WITH SEPARATING MATRIX CONSTRAINT

A. Toeplitz structure constraint

In the above development the last L symbols of $\boldsymbol{d}_k$ are the padded zeros and will not contribute to the observed mixture $\boldsymbol{x}_k$. With $\boldsymbol{d}_k = [\tilde{\boldsymbol{d}}_k^T, 0,...,0\,]^T$, where $\tilde{\boldsymbol{d}}_k$ (of length P) is the transmitted symbol block without padded zeros, and $\tilde{H}$ the truncated version of H containing only the first P columns, we have $\boldsymbol{x}_k = \tilde{H}\tilde{\boldsymbol{d}}_k$. Matrix $\tilde{H}$ is $P\times P$, thus the problem of recovering $\tilde{\boldsymbol{d}}_k$ from $\boldsymbol{x}_k$ can be solved using a standard BSS algorithm. The separating matrix, denoted as $\tilde{B}$, is also $P\times P$.

From the structure of H in (4), we see that $\tilde{H}$ is a square Toeplitz matrix with lower triangular elements zero. Since $\tilde{H}$ is square and full rank, it has a unique left inverse. Thus if the separating matrix is a good approximation of $\tilde{H}^{-1}$ after convergence, the source symbols will be well recovered.

Now it is known that the inverse of a square Toeplitz matrix with lower triangular elements zero has the same structure [8], [9]. Therefore we can modify the EASI algorithm updates by forcing this Toeplitz structure after each update. In the resulting algorithm, which we will call the Toeplitz EASI (T-EASI) algorithm, $\tilde{B}_k$ will be first updated according to (5), then the lower diagonal elements will be set to zero, and the elements on the main and minor diagonals of $\tilde{B}_{k+1}$ will be replaced by the average value of the elements on that diagonal.

Forcing this Toeplitz structure on $\tilde{B}$ will also make the matrix $\tilde{C} \triangleq \tilde{B}\tilde{H}$ a square Toeplitz matrix with lower triangular elements zero.

B. Equalizer length constraint

Consider the elements in $\tilde{H}^{-1}$. Suppose we want to design an equalizer $\boldsymbol{w} = [w(0), w(1),..., w(M)]$ of order M such that the first P taps of the convolution of $\boldsymbol{w}$ and $\boldsymbol{h}$ is the truncated unit-sample sequence. Denote the first P taps of the convolution as $(\boldsymbol{w}*\boldsymbol{h})_{1:P}$, then

$$(\boldsymbol{w}*\boldsymbol{h})_{1:P} = \boldsymbol{c} \qquad (6)$$

with $c(0)=1$, $c(i)=0$ for $i=1,2,...,P-1$. We also assume that $M = P-1$, without loss of generality.

The matrix version of (6) becomes $\boldsymbol{w}\tilde{H} = \boldsymbol{c}$. Since $\tilde{H}$ is a square Toeplitz matrix, we also have

$$[\underbrace{0 \ \ldots \ 0}_{i \text{ zeros}} \ w(0) \ w(1) \ \ldots \ w(M-i)]\tilde{H} = \tilde{\boldsymbol{e}}_i \qquad (7)$$

where $i = 0,1...,M$, and $\tilde{\boldsymbol{e}}_i$ is a length P vector with the i+1-th element one and zero elsewhere (note $\tilde{\boldsymbol{e}}_0 = \boldsymbol{c}$). Denoting the vector on the left side in (7) as $\boldsymbol{w}_i$, we have

$$\left[\boldsymbol{w}_0^T \ \ \boldsymbol{w}_1^T \ \ \ldots \ \ \boldsymbol{w}_M^T\right]^T \tilde{H} = \left[\tilde{\boldsymbol{e}}_0^T \ \ \tilde{\boldsymbol{e}}_1^T \ \ \ldots \ \ \tilde{\boldsymbol{e}}_M^T\right]^T = I \, . \qquad (8)$$

From the above we see that $\tilde{H}^{-1}$ contains the first P=M+1 equalizer coefficients. Matrix $\tilde{W} \triangleq \left[\boldsymbol{w}_0^T \ \boldsymbol{w}_1^T \ \ldots \ \boldsymbol{w}_M^T\right]^T$ is a $P\times P$ Toeplitz matrix with lower triangular elements zero. If $\tilde{C}$ matrix converges ideally to the identity, then $\tilde{B}_k$ converges to $\tilde{W}$, and the rows of $\tilde{B}_k$ matrix contain the impulse response coefficients of the equalizer.

From the analysis in part *A*, we know that the upper rows of the final $\tilde{C}$ will have more non-zero off-diagonal elements (due to imperfect convergence) than the lower ones, and the first several symbols in each transmitted block will therefore be better recovered than the last ones. This is one consequence of the zero-padding. To limit the number of nonzero elements in the top rows, we can force the elements on the uppermost diagonals of matrix $\tilde{B}_k$ to be zero. By doing so, we are adding a constraint that the full equalizer be of particular length (less than P) in each iteration. As a result, the number of possible non-zero elements in each row of matrix $\tilde{C}$ will also be limited to a particular length. Simulation results show that the length-constrained T-EASI scheme (T-LC-EASI) will help improve recovery of the symbols in each block.

IV. BE WITH I/Q CONSTRAINT FOR PHASE RECOVERY

When the source has independent in-phase and quadrature parts, as in the case of standard QAM signaling, N complex source symbols can be seen as $2N$ mutually independent real symbols. In this case, if the sources are well separated, the $2N$ real output symbols of the equalizer should be independent of each other, and the original symbols can be recovered with no phase ambiguity by proper I/Q association.

For simplicity, we ignore noise in (3) and drop the block-index k. Denoting the in-phase and quadrature parts of the channel matrix $\tilde{H}$ as $\tilde{H}_R$ and $\tilde{H}_I$, and those of the signal vector $\tilde{\boldsymbol{d}}$ as $\tilde{\boldsymbol{d}}_R$ and $\tilde{\boldsymbol{d}}_I$, the I and Q components of the observation vector become

$$\boldsymbol{x}_R = \tilde{H}_R\tilde{\boldsymbol{d}}_R - \tilde{H}_I\tilde{\boldsymbol{d}}_I \qquad (9)$$

$$\boldsymbol{x}_I = \tilde{H}_R\tilde{\boldsymbol{d}}_I + \tilde{H}_I\tilde{\boldsymbol{d}}_R \qquad (10)$$

The matrix form of (9) and (10) is

$$\begin{bmatrix}\boldsymbol{x}_R\\ \boldsymbol{x}_I\end{bmatrix} = \begin{bmatrix}\tilde{H}_R & -\tilde{H}_I\\ \tilde{H}_I & \tilde{H}_R\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{d}}_R\\ \tilde{\boldsymbol{d}}_I\end{bmatrix} \qquad (11)$$

The equalizer output $\tilde{\boldsymbol{y}}_k = \tilde{B}_k\boldsymbol{x}_k$ can also be written in terms of

their respective I and Q components, i.e.

$$\begin{bmatrix} \tilde{\boldsymbol{y}}_R \\ \tilde{\boldsymbol{y}}_I \end{bmatrix} = \begin{bmatrix} \tilde{B}_R & -\tilde{B}_I \\ \tilde{B}_I & \tilde{B}_R \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_R \\ \boldsymbol{x}_I \end{bmatrix}. \quad (12)$$

Defining $\bar{H} = \begin{bmatrix} \tilde{H}_R & -\tilde{H}_I \\ \tilde{H}_I & \tilde{H}_R \end{bmatrix}$, $\bar{B} = \begin{bmatrix} \tilde{B}_R & -\tilde{B}_I \\ \tilde{B}_I & \tilde{B}_R \end{bmatrix}$, $\bar{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x}_R \\ \boldsymbol{x}_I \end{bmatrix}$ and $\bar{\boldsymbol{y}} = \begin{bmatrix} \tilde{\boldsymbol{y}}_R \\ \tilde{\boldsymbol{y}}_I \end{bmatrix}$, we have

$$\bar{B}\bar{H} = \begin{bmatrix} \tilde{B}_R\tilde{H}_R - \tilde{B}_I\tilde{H}_I & -\tilde{B}_R\tilde{H}_I - \tilde{B}_I\tilde{H}_R \\ \tilde{B}_R\tilde{H}_I + \tilde{B}_I\tilde{H}_R & \tilde{B}_R\tilde{H}_R - \tilde{B}_I\tilde{H}_I \end{bmatrix}, \quad (13)$$

$$\bar{\boldsymbol{y}} = \bar{B}\bar{\boldsymbol{x}} \ . \quad (14)$$

The original problem becomes: given the observation vector $\bar{\boldsymbol{x}}$, find the separating matrix $\bar{B}$ such that $\bar{\boldsymbol{y}}$ is a good estimation of $\left[\tilde{\boldsymbol{d}}_R^T \ \ \tilde{\boldsymbol{d}}_I^T\right]^T$, i.e. $\bar{B}\bar{H} \simeq I$.

The matrix multiplication $\bar{B}\bar{H}$ consists of four $P \times P$ blocks, of which the diagonal blocks are the same and the other two sum to zero. Thus we want

$$\tilde{B}_R\tilde{H}_R - \tilde{B}_I\tilde{H}_I = I_{P\times P} \quad (15)$$

$$\tilde{B}_R\tilde{H}_I + \tilde{B}_I\tilde{H}_R = \mathbf{0}_{P\times P} \quad (16)$$

The solutions $\tilde{B}_R^{opt}$ and $\tilde{B}_I^{opt}$ satisfying (15) and (16) are

$$\tilde{B}_R^{opt} = \tilde{H}_I^{-1}(\tilde{H}_R\tilde{H}_I^{-1} + \tilde{H}_I\tilde{H}_R^{-1})^{-1} \quad (17)$$

$$\tilde{B}_I^{opt} = -\tilde{H}_I^{-1}(\tilde{H}_R\tilde{H}_I^{-1} + \tilde{H}_I\tilde{H}_R^{-1})^{-1}\tilde{H}_I\tilde{H}_R^{-1} \quad (18)$$

Since $\tilde{H}_R$ and $\tilde{H}_I$ are both square Toeplitz matrices with lower diagonal elements zero, their inverses have the same structure. In addition, since the Toeplitz structure remains under matrix multiplication, $\tilde{B}_R^{opt}$ and $\tilde{B}_I^{opt}$ are also square Toeplitz matrices with lower diagonal elements zero. We see that matrix $\bar{B}$ has a resulting block structure constraint, with identical Toeplitz diagonal blocks and off-diagonal Toeplitz blocks that are sign-inverted versions of each other.

For the I/Q constraint model, we can solve the BE problem using EASI, T-EASI, and T-LC-EASI algorithms; we will label these as the I/Q-EASI, I/Q-T-EASI, and I/Q-T-LC-EASI algorithms, respectively. We specify below the algorithm for I/Q-T-LC-EASI, which includes the steps for I/Q-EASI (steps 1-4) and I/Q-T-EASI (steps 1-5, 7):

Initialization: Initialize $\tilde{B}_0$ with a square Toeplitz matrix with lower diagonal elements zero, get $\bar{B}_0$ from $\tilde{B}_0$.

Serial Updating:

while there is new data $\boldsymbol{x}_k$

1. $\boldsymbol{x}_R = \dfrac{\boldsymbol{x}_k + \boldsymbol{x}_k^*}{2}$, $\boldsymbol{x}_I = \dfrac{\boldsymbol{x}_k - \boldsymbol{x}_k^*}{2j}$, $\bar{\boldsymbol{x}}_k = \left[\boldsymbol{x}_R^T \ \ \boldsymbol{x}_I^T\right]^T$.
2. $\bar{\boldsymbol{y}}_k = \bar{B}_k\bar{\boldsymbol{x}}_k$
3. $\bar{B}_{k+1} = \bar{B}_k - \lambda\left[\dfrac{\bar{\boldsymbol{y}}_k\bar{\boldsymbol{y}}_k^T - I}{1+\lambda\bar{\boldsymbol{y}}_k^T\bar{\boldsymbol{y}}_k} + \dfrac{\boldsymbol{g}(\bar{\boldsymbol{y}}_k)\bar{\boldsymbol{y}}_k^T - \bar{\boldsymbol{y}}_k\boldsymbol{g}^T(\bar{\boldsymbol{y}}_k)}{1+\lambda\,|\,\bar{\boldsymbol{y}}_k^T\boldsymbol{g}(\bar{\boldsymbol{y}}_k)\,|}\right]\bar{B}_k$
4. $\bar{B}_{k+1}^{(lr)} \leftarrow \dfrac{\bar{B}_{k+1}^{(lr)} + \bar{B}_{k+1}^{(ul)}}{2}$, $\bar{B}_{k+1}^{(ul)} \leftarrow \dfrac{\bar{B}_{k+1}^{(lr)} + \bar{B}_{k+1}^{(ul)}}{2}$, $\bar{B}_{k+1}^{(ll)} \leftarrow \dfrac{\bar{B}_{k+1}^{(ll)} - \bar{B}_{k+1}^{(ur)}}{2}$, $\bar{B}_{k+1}^{(ur)} \leftarrow -\dfrac{\bar{B}_{k+1}^{(ll)} - \bar{B}_{k+1}^{(ur)}}{2}$

   where the first superscript letter *l/u* means lower/upper block, and the second letter *l/r* means left/right block.
5. Set to zero the lower diagonal elements of $\bar{B}_{k+1}^{(lr)}$, $\bar{B}_{k+1}^{(ll)}$, $\bar{B}_{k+1}^{(ur)}$, $\bar{B}_{k+1}^{(ul)}$.
6. Set to zero the upper $P-M-1$ diagonal elements of $\bar{B}_{k+1}^{(lr)}$, $\bar{B}_{k+1}^{(ll)}$, $\bar{B}_{k+1}^{(ur)}$, $\bar{B}_{k+1}^{(ul)}$.
7. $(\bar{B}_{k+1}^{(lr)})_{ij} \leftarrow \dfrac{1}{P-(j-i)}\sum_{m-s=i-j}(\bar{B}_{k+1}^{(lr)})_{ms}$, $i,j,m,s = 1,2...P$, $j \geq i$. Repeat for $\bar{B}_{k+1}^{(ll)}$, $\bar{B}_{k+1}^{(ur)}$ and $\bar{B}_{k+1}^{(ul)}$.

end while

Signal Estimation: The first *P* elements of $\bar{\boldsymbol{y}}$ give the real parts of the estimated sources, and the next *P* elements give the imaginary parts.

V. CONTINUOUS SEQUENCE TRANSMISSION

When symbols are transmitted in blocks with zeros padded, ISI will happen only within blocks, which makes it possible to formulate a standard BSS problem. However, the block transmission scheme may need a large number of symbol blocks for convergence, and the padded zeros reduce the effective symbol rate. In this section we show how BE can be accomplished with source symbols transmitted continuously.

In the continuous transmission case, the observation vectors $\{\boldsymbol{x}_t\}$ are formed by sliding along the observation sequence, and *P* observation symbols will be processed at each time. Defining $\boldsymbol{x}_t = [x(t), x(t-1), ..., x(t-P+1)]^T$ and source vector $\boldsymbol{d}_t = [a(t), a(t-1), ...a(t-N+1)]^T$, where $N = P + L$ and $a(t)$ is the symbol from the source at time *t*, we have $\boldsymbol{x}_t = H\boldsymbol{d}_t$.

In the continuous scheme, we want to design an equalizer $\boldsymbol{w} = [w(0), w(1), ..., w(M)]$ such that the convolution of $\boldsymbol{w}$ and $\boldsymbol{h}$ is approximately the truncated unit-sample sequence of length *M+L+1*, i.e.

$$\boldsymbol{w} * \boldsymbol{h} \simeq \boldsymbol{r} \quad (19)$$

with $r(0) = 1$, $r(i) = 0$ for $i = 1,2,...,M+L$.

We construct a $(P-M)\times P$ Toeplitz matrix *W* from the equalizer vector $\boldsymbol{w}$, with the same structure as in (4). This gives

$$WH \simeq [\boldsymbol{e}_0^T \ \ \boldsymbol{e}_1^T \ \ ... \ \ \boldsymbol{e}_{P-M-1}^T]^T \quad (20)$$

of size $(P-M)\times(P+L)$, where $\boldsymbol{e}_i$ is a length *P+L* row vector with the *i*+1-th element one and zero elsewhere (note $\boldsymbol{e}_0 = \boldsymbol{r}$). Suppose a $(P-M)\times P$ matrix B_t converges to *W*, then with $\boldsymbol{y}_t = B_t\boldsymbol{x}_t = B_tH\boldsymbol{d}_t \rightarrow WH\boldsymbol{d}_t$ we will be able to get the first *P-M* elements of $\boldsymbol{d}_t$. To get the matrix *B*, we can use the EASI and T-EASI algorithms to do the updates. In the T-EASI

algorithm for the continuous case, the Toeplitz structure constraint is imposed so that the separating matrix W satisfies the Toeplitz structure as in (4). The length M+1 of the equalizer is implicitly a constraint included in the updates.

In this continuous transmission scenario, the first $P-M$ source symbol will be recovered. Since these output components are estimates from a sliding block, with different time lags, we only need to pick one specific time-lag to recover the source sequence. Different criteria are suggested in [4] to select the best equalized source sequence. In our simulations we select the output sequence with maximum kurtosis. As in the case of block transmission, we can use the I/Q scheme of section IV under continuous transmission for phase recovery.

VI. Simulation

All our simulations are done for a complex channel and 64-QAM source.

We will first demonstrate performance of EASI, T-EASI and T-LC-EASI algorithms for the block transmission scheme. Consider a 64-QAM source transmitted in blocks with padded zeros through an FIR channel with AWGN and SNR=30dB. The source is normalized so that $E[\boldsymbol{s}\boldsymbol{s}^T]=I$. The order of the channel is $L=6$, and its impulse response is shown in Fig. 1. For symbol block size $P=20$, the size of each transmitted symbol block with padded zeros is $N=26$. In all simulations $T=4000$ blocks were transmitted, so the total number of symbols (excluding the padded zeros) is 80000. To obtain convergence within 3500 blocks, the step-sizes for different algorithms were chosen as follows: $\lambda_{EASI}=0.0004$, $\lambda_{T-EASI}=0.0007$, $\lambda_{T-LC-EASI}=0.002$, $\lambda_{I/Q-EASI}=0.0005$, $\lambda_{I/Q-T-EASI}=0.0007$, $\lambda_{I/Q-T-LC-EASI}=0.002$. For all algorithms the separating matrix was initialized with $(1+0.5j)I_{P\times P}$. The nonlinear function in the algorithms was the phase preserving cubic, i.e. $g(x)=|x|^2 x$. For T-LC-EASI and I/Q-T-LC-EASI, the order of the equalizer was $M=8$. The ISI of each row of matrix C was computed in each iteration, where the ISI of the i-th row is defined as

$$\mathrm{ISI}_i=\sum_{j=1}^{P}\frac{|C_{ij}|^2}{\max_j |C_{ij}|^2}-1 \qquad (21)$$

The sequence of average ISI over all rows of matrix C shows how well the source symbols are recovered.

We also plot the absolute values of the coefficients of C to judge convergence performance. Each experiment was run 20 times and the ISI profiles displayed are the average over these 20 runs. The coefficients of the C matrix are from the last run.

In Fig. 2, the absolute value of the coefficients of matrix C with EASI, T-EASI and T-LC-EASI are shown. From the figure, we can see that the absolute value of the coefficients of C matrix with T-EASI concentrate more around zero and one than with EASI, and the convergence is faster. T-LC-EASI gives better performance than T-EASI.

Fig. 3 gives the average ISI of the C matrix with EASI, T-EASI, T-LC-EASI and their I/Q version. From the figure, it can be seen that convergence speed with and without I/Q constraint is almost the same. (I/Q-)T-EASI and (I/Q-)EASI both need about 3300 blocks (66000 symbols) to converge, but (I/Q-)T-EASI gives much lower ISI (-35dB) than (I/Q-)EASI (-23dB) after convergence. (I/Q-)T-LC-EASI will reach the same ISI after convergence as (I/Q-)T-EASI, while it needs only about 1000 blocks (20000 symbols) to converge.

Fig. 4 gives the constellation of the output signal. We see that the plots of output for (I/Q-)T-EASI and (I/Q-)T-LC-EASI are more concentrated than for (I/Q-)EASI, which is consistent with the ISI plots in Fig. 3. With the I/Q constraint, the sources can be recovered without phase ambiguity, as is clear from Fig. 4.

In the second experiment, a 64-QAM source is transmitted continuously with no zero padding. The source, channel, and SNR is the same as before. The step-sizes are chosen as $\mu_{EASI}=\mu_{I/Q-EASI}=0.00015$, $\mu_{T-EASI}=\mu_{I/Q-T-EASI}=0.00026$, so that all algorithms take less than 18000 symbols to converge. The parameters are chosen as $M=8$, $P=12$, and the separating matrix is initialized as $B_0=(1+0.25j)\left[I_{4\times 4}\,|\,\mathbf{0}_{4\times 8}\right]$.

After convergence we pick the best row of output components with maximum kurtosis, and the ISI of that row during the entire updating process is displayed.

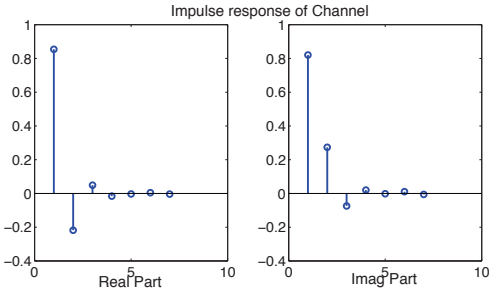

Fig. 1 Impulse response of the channel.

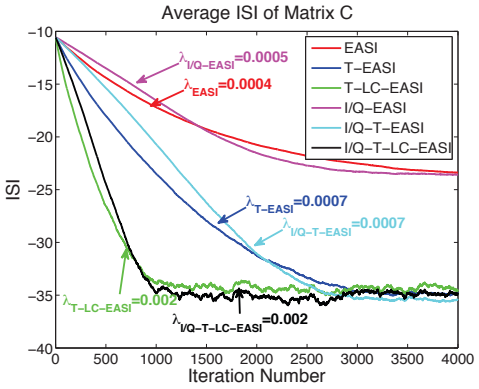

Fig. 3 Average ISI of rows of C when the source is 64-QAM, SNR=30dB. $P=20$, $M=8$.

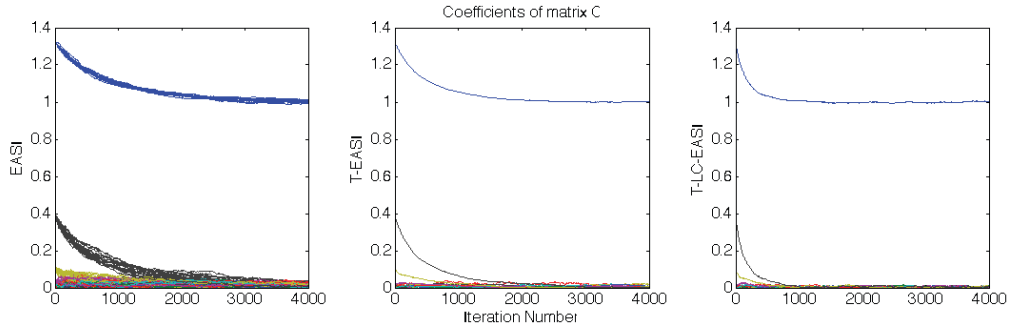

Fig. 2 Magnitude of coefficients of C matrix when the source is 64-QAM under SNR=30dB. $P = 20$, $M = 8$.

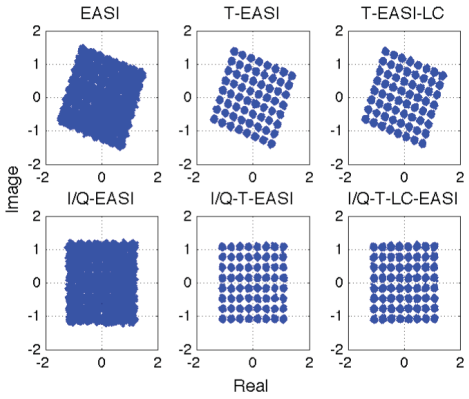

Fig. 4 Plots of the recovered sources for block transmission.

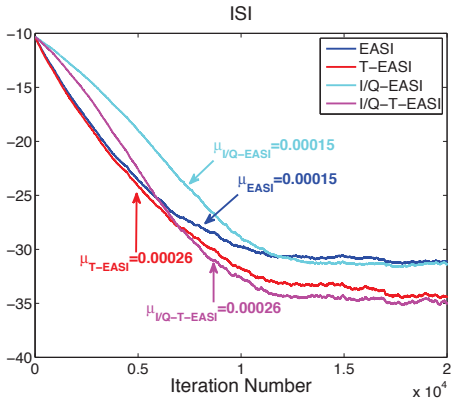

Fig. 5 ISI of the best row of C matrix when the source is 64-QAM, SNR=30dB. $P = 12$, $M = 8$.

In Fig. 5, we compare the ISI of the best row with (I/Q-)EASI and (I/Q-)T-EASI. From the figure, it can be seen that with the Toeplitz structure constraint, ISI can be decreased to −35dB, compared with −31.5dB using standard EASI algorithm. The recovered signals are plotted in Fig. 6, which illustrates the phase recovery with I/Q constraint.

Comparing the block transmission scheme and continuous transmission scheme, we can see that the latter gives faster convergence. The ISI after convergence for T-LC-EASI in block transmission scheme and T-EASI in continuous scheme are the same (-35dB), but the continuous scheme gives better constellation recovery.

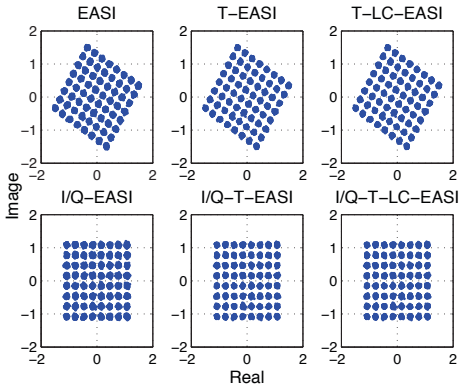

Fig. 6 Plots of the recovered sources for continuous transmission.

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